

it far more difficult to see. For this reason I have never been able to obtain in England satisfactory measures of distance of these inner satellites.

Dec. 18. Definition very bad and measures most difficult to take.

The instrument used was the 20-feet Equatorial with a magnifying power, generally, of 614.

Bradstones, near Liverpool, 4th April, 1857.

Erratum.—In the *Monthly Notices*, vol. xiii. p. 149, the position of *Oberon*, Nov. 2, instead of $206^{\circ} 40'$ should be $196^{\circ} 40'$. This error was discovered by Dr. Marth, who, in discussing these observations found this measure to stand out greatly from the series; and on reference to my MSS., I find it occurred in an erroneous addition in deducing the angle from the north point from the angle with the parallel.

Note to the Paper on Optical Phenomena in Occultations in the preceding Number. By Professor Powell.

“In my remarks on the observations, I put a suggestion whether the appearance observed by Prof. Challis might not have been a projection. I have since received a letter from that gentleman, in which he states, ‘The depression which I noticed at the part of moon’s limb when *Jupiter* reappeared was an actual inequality of the limb, and was as conspicuous before and after, as during the occultation.’

“He further observes that what he saw agrees well with Mr. Hartnup’s description (*Monthly Notices*, vol. xvii. p. 73). He also saw the second satellite, at its reappearance, bisected by the moon’s limb just at the extremity of the same depression.

“I am anxious to take the earliest opportunity of correcting any misapprehension on this point.”

On the Correction of Sextant-Observations. By W. H. Simms, Esq.

In two papers recently laid before the Royal Astronomical Society I have discussed in detail the errors to which the several parts of the sextant are liable, and the effects of these errors upon observations made with that instrument. But as in the said papers the subject has been treated at some length, and as practical men, scientific explorers and navigators, who are in the habit of using the sextant, will be glad to have it placed before them in a simple and practical form, I omit on this occasion everything of the nature of investigation, confining myself to formulæ and examples, a careful consideration of which will enable any person tolerably well versed in the manipulation of tables to correct his observations, and thus to obtain results upon which he may rely to a much greater extent than he could upon such as he might obtain

from the simple readings corrected only for index-error in the ordinary way.

The principal error for which correction is required is that of eccentricity; an error from which few sextants are exempt, and which affects unequally the angles read from different points of the limb, the maximum effect being seldom less than thirty seconds, sometimes exceeding two minutes. The formula for correction reduced to its most simple form, and including the effect of index-error is

$$\Omega = \omega + \varepsilon + \frac{1}{2}k \cdot \sin \left\{ \nu + \frac{1}{2}(\omega) \right\}^*$$

where ω is the reading made in the ordinary way; (ω) the degrees and minutes on the arc at the point of coincidence of divisions; ε, k, ν , constants; and Ω the corrected angle. By means of this formula, the values of the constants being known, any reading may be corrected; or we may form a table of corrections for every 5 or 10 degrees of the arc, and thence take the particular correction required on each several occasion.

To find the values of the constants, three known angles must be observed, one near to each extremity, the third near the middle of the arc. Let the known values of these angles be, in order of magnitude, $\Omega_1, \Omega_2, \Omega_3$; and let $\omega_1, \omega_2, \omega_3$, be the uncorrected readings from the limb of the sextant; $(\omega_1), (\omega_2), (\omega_3)$, the degrees and minutes on the limb at the points of coincidence of divisions. Compute

$$c_1 = \Omega_2 - \Omega_1 - (\omega_2 - \omega_1) \quad c_2 = \Omega_3 - \Omega_1 - (\omega_3 - \omega_1)$$

$$\theta_1 = \frac{1}{4} \{ (\omega_2) - (\omega_1) \}; \quad \theta_1' = \frac{1}{4} \{ (\omega_2) + (\omega_1) \};$$

$$\theta_2 = \frac{1}{4} \{ (\omega_3) - (\omega_1) \}; \quad \theta_2' = \frac{1}{4} \{ (\omega_3) + (\omega_1) \}.$$

$$\tan \nu = \frac{c_1 \cdot \sin \theta_2 \cdot \cos \theta_2' - c_2 \cdot \sin \theta_1 \cdot \cos \theta_1'}{c_1 \cdot \sin \theta_2 \cdot \sin \theta_2' - c_2 \cdot \sin \theta_1 \cdot \sin \theta_1'};$$

$$k = \frac{c_1}{\sin \theta_1 \cdot \cos (\nu + \theta_1')};$$

$$\varepsilon = \Omega_1 - \omega_1 - \frac{1}{2} \cdot k \cdot \sin \left\{ \nu + \frac{1}{2}(\omega_1) \right\}$$

whence the constants are determined.

We may now for every 5° or 10° of (ω) compute and tabulate the values of $\frac{1}{2}k \cdot \sin \left\{ \nu + \frac{1}{2}(\omega) \right\}$; and the process of correcting any angle ω will then consist in the application, first of ε , then of the value of $\frac{1}{2}k \cdot \sin \left\{ \nu + \frac{1}{2}(\omega) \right\}$ standing opposite to (ω) in the table.

It is necessary only that the index-error should not change in the course of the observations from which we derive the values of the constants; but it may vary afterwards, and as a variation in this will produce one in the value of ε , we shall have to determine ε from time to time in the following manner:—

Let ω, ω' be the two readings which correspond to opposite contact of the sun's limb; $(\omega), (\omega')$, the degrees and minutes at

* ν is the angle ZOY, and ε the quantity $\omega' - \frac{1}{2} \cdot k \cdot \sin \text{ZOA}'$ of the investigation contained in the second of the papers laid before the Society.

coincidence of divisions. Then, $2r$ being the sun's apparent diameter,

$$\begin{aligned} 2r &= \omega + \varepsilon + \text{quantity opposite } (\omega) \text{ in the Table} \\ -2r &= \omega' + \varepsilon + \text{quantity opposite } (\omega') \text{ in the Table} \end{aligned}$$

and therefore $\varepsilon = -\frac{1}{2}(\omega + \omega') - \frac{1}{2}$ sum of quantities opposite (ω) and (ω') in the table.

With respect to the three known angles to be observed for the determination of the constants; of these, the first may be either the sun's apparent diameter, or actual coincidence of the two images of a star, in which latter case the angle will be 0. The second and third should be distances of well-known stars from one another, these distances being computed for the occasion, or circum-meridian altitudes at a place of which the latitude is known. Each observation should of course be repeated several times.

To compute the apparent distance between two stars, we have the following formula:—

Let π_1, A_1 be the true polar distance and right ascension of the first star; π_2, A_2 of the second;

ζ_1, ζ_1' the apparent and true zenith distance of the first; ζ_2, ζ_2' of the second;

the polar distances and right ascensions being taken from the tables; and the zenith distances either computed, or derived from altitudes observed in the course of the observations of distance.

Compute

$$\begin{aligned} \log \tan \theta &= \frac{1}{2} \{ \log \sin \pi_1 + \log \sin \pi_2 \} \\ &\quad + \log \sin \frac{1}{2} (A_1 \sim A_2) \} - \log \sin \frac{1}{2} (\pi_1 \sim \pi_2); \\ \log \sin \frac{1}{2} D &= \log \sin \frac{1}{2} (\pi_1 \sim \pi_2) - \log \cos \theta; \\ \beta &= \frac{1}{2} (\zeta_1' + \zeta_2' + D) \\ \log \sin \psi &= \frac{1}{2} \{ \log \sin \beta + \log \sin (\beta - D) + \log \sin \zeta_1 + \log \sin \zeta_2 \\ &\quad - (\log \sin \zeta_1' + \log \sin \zeta_2') \} - \log \sin \frac{1}{2} (\zeta_1 + \zeta_2) \end{aligned}$$

Then, D' being the apparent distance required,

$$\log \sin \frac{1}{2} D' = \log \sin \frac{1}{2} (\zeta_1 + \zeta_2) + \log \cos \psi.$$

The following is an example of the process:—

Suppose that on the 1st of March, 1857, in latitude $51^\circ 30' N.$, and the longitude of Greenwich, the observations were

1. Mean of ten readings for contact of sun's limb = $+30' 11''$; the sun's apparent altitude being 30° , and the Bar. $30^{\text{in}}.00$, Ther. 54° .

2. At $6\frac{1}{2}^{\text{h}}$ mean time. Mean of ten observed distances between *Capella* and α *Orionis* = $39^\circ 27' 5''$.

3. At 7^{h} mean time. Mean of ten observed distances between α *Ursæ Majoris* and *Sirius* = $93^\circ 17' 23''$.

The barometer and thermometer for observations 2 and 3 being $29^{\text{in}}.95$ and 40° .

On the 1st of March the sun's true diameter = $32' 20''$; and the difference of refraction for $32'$ at altitude 30° is $1''$. The

apparent vertical diameter therefore = $32' 19''$; and the first observation gives

$$\Omega_1 = 32' 19''; \quad \omega_1 = 30' 11''; \quad (\omega_1) = 0^\circ 41'.$$

Next to compute the apparent distance between α *Orionis* and *Capella* at $6\frac{1}{2}^h$ P.M.

A_1	$= 5^h 47^m 26\cdot64^s$	π_1	$= 82^\circ 37' 21''$
A_2	$= 5 \quad 6 \quad 8\cdot53$	π_2	$= 44 \quad 8 \quad 52$
$\frac{1}{2}(A_1 - A_2)$	$= 20 \quad 39\cdot06$	$\frac{1}{2}(\pi_1 - \pi_2)$	$= 19 \quad 14 \quad 15$
or	$= 5^\circ 9' 46''$		

The altitudes were not observed. We must therefore compute the zenith-distances from the formulæ,

$$\log \tan \chi = \log \tan \phi + \log \cos H;$$

$$\log \cos \zeta' = \log \cos \phi - \log \cos \chi + \log \cos (\pi - \chi);$$

where ϕ is the colatitude of the place, and H the hour-angle of the star from the meridian.

Log tan ϕ	$= 9\cdot9006052$	Log cos ϕ	$= 9\cdot8935444$
Log cos H_1	$= 9\cdot9935398$	Log cos χ	$= 9\cdot8960247$
Log tan χ	$= 9\cdot8941450$	Diff.	$= 9\cdot9975197$
χ	$= 38^\circ 5' 8''$	Log cos $(\pi_1 - \chi)$	$= 9\cdot8529667$
π_1	$= 82 \quad 37 \quad 21$	Log cos ζ_1'	$= 9\cdot8504864$
$\pi_1 - \chi$	$= 44 \quad 32 \quad 13$	ζ_1'	$= 44^\circ 52' 3''$
		Refraction	$= -0 \quad 59$
		ζ_1	$= 44 \quad 51 \quad 4$

Similarly we shall obtain $\zeta_2' = 5^\circ 39' 19''$, $\zeta_2 = 5^\circ 39' 13''$.

Log sin π_1	$= 9\cdot9963898$	Log sin $\frac{1}{2}(\pi_1 - \pi_2)$	$= 9\cdot5178352$
Log sin π_2	$= 9\cdot8429282$	Log cos θ	$= 9\cdot9890911$
Sum	$= 9\cdot8393180$	Log sin $\frac{1}{2} D$	$= 9\cdot5287441$
Half	$= 9\cdot9196590$	$\frac{1}{2} D$	$= 19^\circ 44' 49''$
Log sin $\frac{1}{2}(A_1 - A_2)$	$= 8\cdot9541729$		
Sum	$= 8\cdot8738319$	D	$= 39 \quad 29 \quad 38$
Log sin $\frac{1}{2}(\pi_1 - \pi_2)$	$= 9\cdot5178352$	$\therefore \beta$	$= 45 \quad 0 \quad 30$
Log tan θ	$= 9\cdot3559967$	$\beta - D$	$= 5 \quad 30 \quad 52$

* This may be either taken from the arc at the time of reading, or derived subsequently by adding to the degrees of ω the number of minutes taken from the vernier, and to the minutes of ω taken from the arc the number of seconds taken from the vernier.

Log sin β	= 9.8495482	Log sin ζ_1'	= 9.8484783
Log sin ($\beta - D$)	= 8.9827084	Log sin ζ_2'	= 8.9936259
Log sin ζ_1	= 9.8483535	K	= 8.8421042
Log sin ζ_2	= 8.9934983		
Sum	= 7.6741084	Log sin $\frac{1}{2}(\zeta_1 + \zeta_2)$	= 9.6300292
K	= 8.8421042	Log cos ψ	= 9.8985635
Diff.	= 8.8320042	Log sin $\frac{1}{2}D'$	= 9.5285927
Half	= 9.4160021		
Log sin $\frac{1}{2}(\zeta_1 + \zeta_2)$	= 9.6300292	$\frac{1}{2}D'$	= 19° 44' 23"
Log sin ψ	= 9.7859729	D'	= 39 28 46

Hence the second observation gives

$$\Omega_2 = 39^\circ 28' 46''; \quad \omega_2 = 39^\circ 27' 5''; \quad (\omega_2) = 46^\circ 25'.$$

Similarly from the third we derive

$$\Omega_3 = 93^\circ 18' 43''; \quad \omega_3 = 93^\circ 17' 23''; \quad (\omega_3) = 100^\circ 33'.$$

Hence,

$$\begin{aligned} \Omega_2 - \Omega_1 &= 38^\circ 56' 27''; \quad \omega_2 - \omega_1 = 38^\circ 56' 54''; \quad c_1 = -27''; \\ \Omega_3 - \Omega_1 &= 92^\circ 46' 24''; \quad \omega_3 - \omega_1 = 92^\circ 47' 12''; \quad c_2 = -48''; \\ (\omega_2) - (\omega_1) &= 45^\circ 44'; \quad (\omega_2) + (\omega_1) = 47^\circ 6'; \quad \theta_1 = 11^\circ 26'; \quad \theta_1' = 11^\circ 47'; \\ (\omega_3) - (\omega_1) &= 99^\circ 52'; \quad (\omega_3) + (\omega_1) = 101^\circ 14'; \quad \theta_2 = 24^\circ 58'; \quad \theta_2' = 25^\circ 19'. \end{aligned}$$

Log $-c_1$	= 1.43136	Log $-c_1$	= 1.43136
Log sin θ_2	= 9.62541	Log sin θ_2	= 9.62541
Log cos θ_2'	= 9.95615	Log sin θ_2'	= 9.63106
Sum	= 1.01292	Sum	= 0.68783
Nat. No.	10.302	Nat. No.	4.873

Log $-c_2$	= 1.68124	Log $-c_2$	= 1.68124
Log sin θ_1	= 9.29716	Log sin θ_1	= 9.29716
Log cos θ_1'	= 9.99075	Log sin θ_1'	= 9.31008
Sum	= 0.96915	Sum	= 0.28848
Nat. No.	9.314	Nat. No.	1.943

Whence,

$$\begin{aligned} \tan \nu &= \frac{-10.302 + 9.314}{-4.873 + 1.943} = \frac{0.988}{2.930}. \quad \text{Log tan } \nu = 9.52789; \\ \nu &= 18^\circ 38'; \quad \nu + \theta_1' = 30^\circ 25'; \quad \nu + \frac{1}{2}(\omega_1) = 19^\circ 0'. \end{aligned}$$

$$\begin{array}{rcl}
 \text{Log cos } (\nu + \theta_1') & = & 9.93569 \\
 \text{Log sin } \theta_1 & = & 9.29716 \\
 \hline
 \text{Sum} & = & 9.23285 \\
 \text{Log } -c_1 & = & 1.43136 \\
 \hline
 \text{Log } -k & = & 2.19851 \\
 \text{Log sin } \left\{ \nu + \frac{1}{2} (\omega_1) \right\} & = & 9.51264 \\
 \hline
 & & 1.71115 = \text{Log } 51''.4
 \end{array}$$

Therefore $\epsilon = 2' 8'' + 25''.7 = + 2' 33''.7$; and the general equation is

$$\Omega = \omega + 2' 33''.7 - 78''.97 \cdot \sin \left\{ 18^\circ 38' + \frac{1}{2} (\omega) \right\}.$$

We now compute the following table of the values of

$$\frac{1}{2} k \cdot \sin \left\{ \nu + \frac{1}{2} (\omega) \right\} :$$

(ω)		(ω)		(ω)	
0	$-25''.2$	35	$-46''.6$	70	$-63''.6$
5	28.5	40	49.3	75	65.6
10	31.7	45	51.9	80	67.4
15	34.8	50	54.5	85	69.2
20	37.8	55	56.9	90	70.8
25	40.8	60	59.3	95	72.2
30	43.7	65	61.5	100	73.5

and we are thus furnished with all we require for the correction of angles.

Suppose, for example, it were required to correct the angle $39^\circ 27' 5''$ read from the limb of the sextant. Here,

$$\omega = 39^\circ 27' 5'', (\omega) = 39^\circ 20' + 7^\circ 5' = 46^\circ 25'$$

and the corrected angle,

$$= 39^\circ 27' 5'' + 2' 33''.7 - 52''.9 = 39^\circ 28' 45''.8.$$

A table similar to the above will continue to answer during a long period for the instrument for which it was obtained, the values of the constants k and ν not being liable to disturbance except in consequence of serious accident or the operations of the maker in the course of repair. The constant ϵ is however liable to occasional disturbance, and its value must therefore be determined from time to time in the manner already explained. As an example we will suppose that on the 10th of March the following were the observations made for the purpose of a new determination of its value :—

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Reading corresponding to positive contact of $\odot$'s limb	$=\omega = +30^{\circ} 16''$	$\omega' = 0^{\circ} 46''$
negative	$=\omega' = -34^{\circ} 8''$	$(\omega') = 5^{\circ} 12''$
Quantity opposite (ω) in the table	$= -25^{\circ} 7''$	$\omega + \omega' = -3^{\circ} 52''$
(ω')	$= -28^{\circ} 6''$	Half $= -1^{\circ} 56''$
	$= -54^{\circ} 3''$	$= -27^{\circ} 2''$
		$\epsilon = -2^{\circ} 23^{\circ} 2''$

and this new value of ϵ must now be applied to all angles alike, the second correction being taken from the table as before.

When the sun is observed for the purpose of determining the values of the constants, the image should be darkened by a shade placed before the eye-piece of the telescope.

The value of ϵ will in most cases be more or less affected by the interposition of shades before the index-glass, or behind the horizon-glass. The corrections due to each particular shade should therefore be determined, in order that in the event of our employing them for the purpose of darkening the image of the sun or moon in the course of ordinary observation, we may be prepared to apply the proper constant in the correction of the angle. This is done by observing, first, the contact of the images of the sun's limb with a shade before the eye-piece of the telescope only. The same contact being then observed with as many different combinations of the other shades as their number amounts to, we shall be furnished with as many simple equations, from which the several corrections may be determined.

A traveller on land treating the sextant in the manner above described ought very readily to obtain his latitude to the nearest second of arc: and if the results of his lunar distances be subsequently corrected for the errors in the tabular elements of the moon's place, his longitudes will probably be in most instances much within ten seconds of time from the truth. And as regards lunar observations at sea, the results must of course be materially affected by the correction of the observed angles to an extent exceeding in general the amount of probable error arising, even under unfavourable circumstances, from the difficulties attending the operation, and from deficiency of power in the instrument.

On the Value of the Constant of Refraction, as determined from Zenith-Distance Observations of Stars near the North and South Horizon, made at the Royal Observatory, Greenwich, in the years from 1836 to 1854. By the Rev. Robert Main, M.A.

The object of this paper is stated by the author to be the determination of the degree of accuracy with which Bessel's Refraction Tables, given in the *Tabulæ Regiomontanæ*, represent the star-observations made at Greenwich. Those tables, as is well known,